

# FIRST SEMESTER GEO w/ Proof Review

1. Given that quadrilateral  $ABCD$  in Figure 4 is a parallelogram, which of the following statements would lead to the conclusion that  $ABCD$  is a rectangle?

- (A)  $AC$  and  $BD$  are the perpendicular bisectors of each other
- (B)  $AB = CD$
- (C)  $AB = BC$
- (D)  $\angle DAB$  and  $\angle CBA$  are supplementary
- (E)  $\angle DAB$  and  $\angle CBA$  are equal in measure

2. A set of points is said to "determine" a plane if there is exactly one plane that contains all of the points. Which of the following does not determine a plane?

- (A) A triangle
- (B) Any three points
- (C) A line and a point not on the line
- (D) A pair of intersecting rays
- (E) A pair of intersecting lines

3. If  $A$  and  $B$  are two different points that lie on line  $CD$  and also on line  $EF$ , which of the following statements must be true?

- I. Lines  $AC$ ,  $BF$ , and  $ED$  are all parallel.
- II. Points  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  all lie on the same line.
- III. Points  $A$ ,  $B$ ,  $C$ , and  $E$  all lie in the same plane.

- (A) I only
- (B) I and II only
- (C) II only
- (D) II and III only
- (E) III only

4. If  $A$ ,  $B$ ,  $C$ , and  $D$  are any four distinct points of a plane  $P$ , which of the following must be true?

- I. Line  $AB$  must intersect line  $CD$ .
- II. Line  $AB$  can contain a point not in plane  $P$ .
- III. Any point of intersection of any of the pairs of lines containing  $A$ ,  $B$ ,  $C$ , and  $D$  must lie in plane  $P$ .

- (A) I only
- (B) I and II only
- (C) II and III only
- (D) II only
- (E) III only

5. In Figure 2, lines  $L_1$  and  $L_2$  are NOT parallel and will intersect at some point to the right of the page. Which of the following must be true?

- I.  $c = e$
- II.  $b > d$
- III.  $a = c$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) II and III only

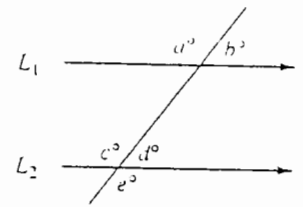


Figure 2

6. If  $A = (-1, 2)$  and  $B = (2, -1)$ , where  $A$  and  $B$  are two points in the coordinate plane, then what is the length of segment  $AB$ ?

- (A) 1
- (B) 2
- (C) 3
- (D) 4.24
- (E) 3.46

7. An operation,  $\circ$ , on the numbers  $a$  and  $b$  is defined by the formula  $a \circ b = 2(a + 2b)$ . For what values of  $x$  and  $y$  is  $x \circ y = y \circ x$ ?

- (A) All real values
- (B) Only when  $x = y$
- (C) Only when both  $x$  and  $y$  are 0
- (D) Only when  $x = -y$
- (E) Only when  $x$  and  $y$  are both 1

8. Which of the following statements about angles is NOT true?

- (A) An angle of a triangle may have a degree measure of 180.
- (B) An angle of a triangle may not have a degree measure greater than 180.
- (C) Two angles may be complementary without having a common side.
- (D) The degree measure of an angle does not depend on the lengths of its sides.
- (E) A triangle cannot have two right angles

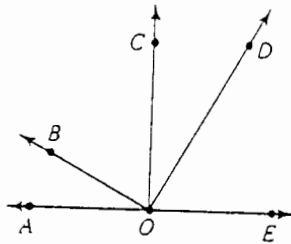
9. If (1) point  $P$  is in the interior of  $\angle LMN$ , (2) point  $W$  is in the interior of  $\angle XYZ$ , (3) the measures of  $\angle LMN$  and  $\angle XYZ$  are equal, and (4) the measures of  $\angle PMN$  and  $\angle WYZ$  are equal, which of the following must be true?

- I.  $\angle LMP$  and  $\angle PMN$  are adjacent angles.
- II. The measure of  $\angle LMN$  equals the sum of the measures of  $\angle LMP$  and  $\angle PMN$ .
- III. The measure of  $\angle ZYW =$  measure of  $\angle LMP$ .

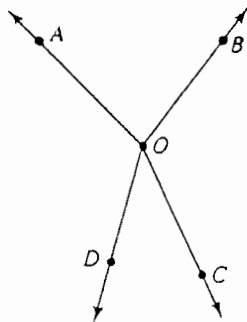
- (A) I only
- (B) II only
- (C) I and II only
- (D) II and III only
- (E) I and III only

10. If in the figure  $\angle COE$  and  $\angle BOD$  are right angles and the measure of  $\angle BOC$  is three times the measure of  $\angle COD$ , then the measure of  $\angle AOB$  is

- (A)  $20^\circ$
- (B)  $22\frac{1}{2}^\circ$
- (C)  $30^\circ$
- (D)  $45^\circ$
- (E)  $60^\circ$



11. If  $\angle AOB$  is a right angle, decide whether you can conclude that  $\angle AOD$  and  $\angle BOC$  have the same measure. Which of the following justifies your conclusion?

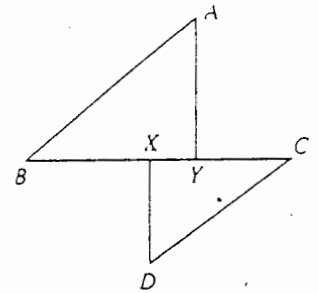


(Figure is not drawn to scale.)

- (A) This conclusion cannot be justified from the information given.
- (B) Complements of congruent angles are congruent.
- (C) Supplements of congruent angles are congruent.
- (D) Vertical angles are congruent.
- (E) If two intersecting lines form one right angle, they must form four right angles.

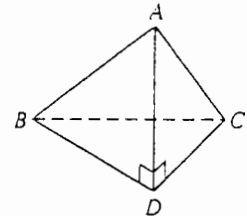
12. In the figure  $\angle AYB$  and  $\angle CXD$  are right angles.  $AY = CX$ , and  $BX = DX = CY$ . Each of the following is true EXCEPT

- (A)  $BX + XY = BY$
- (B)  $XY + CY = CX$
- (C)  $XY = BY - BX = CX - CY$
- (D)  $BX = CX$
- (E)  $BY = CX$



(Figure is not drawn to scale.)

13. If, in the accompanying figure,  $AD \perp DC$ ,  $AD \perp BD$ , and  $DC = BD$ , which of the following statements is NOT necessarily true?

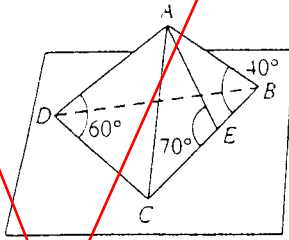


- (A)  $\angle DBC$  and  $\angle DCB$  are equal in measure.
- (B)  $\triangle ADB \cong \triangle ADC$ .
- (C)  $AB = AC$ .
- (D)  $\angle ABC$  and  $\angle ACB$  are equal in measure.
- (E)  $\angle BAC$  and  $\angle BDC$  have the same measure.

14. If  $A$ ,  $B$ , and  $C$  are any three points of a plane for which  $AB = 5$  and  $BC = 7$ , which of the following can be the length of  $AC$ ?

- (A) 0
- (B) 1
- (C) 2
- (D) 13
- (E) 15

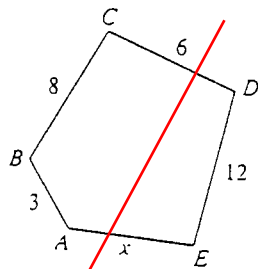
15. Segment  $AC$  is perpendicular to plane  $E$ . The angles have the measures indicated while  $\angle ADB$  and  $\angle ABD$  have measures 70 and 50 respectively. If segments  $AD$ ,  $AC$ ,  $AE$ , and  $AB$  are arranged in order from shortest to longest, which of the following is the result?



- (A)  $AE, AD, AB, AC$   
 (B)  $AE, AB, AC, AD$   
 (C)  $AC, AB, AE, AD$   
 (D)  $AC, AB, AD, AE$   
 (E)  $AC, AE, AD, AB$

16. For a pentagon, 5 is the longest side and 2 is the shortest. Which of the following could not be the length of a diagonal?

- (A) 3 *HINT: A Pentagon has 5 sides.*  
 (B) 4  
 (C) 5  
 (D) 7  
 (E) 11

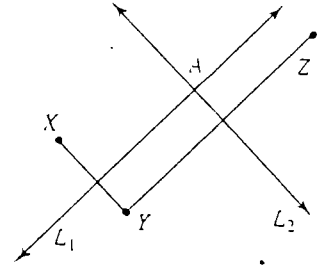


A pentagon has sides of lengths shown in the figure. The range of possible values of  $x$  is?

- (A)  $0 < x < 29$   
 (B)  $11 < x < 18$   
 (C)  $3 < x < 12$   
 (D)  $6 < x < 8$   
 (E)  $0 < x < 20$

18. If  $L_1$  and  $L_2$  are the perpendicular bisectors of segments  $XY$  and  $YZ$ , which of the following must be true?

- I.  $AX = AY = AZ$   
 II.  $A, X$ , and  $Z$  are collinear  
 III.  $L_1$  and  $L_2$  lie in the plane determined by  $X, Y$ , and  $Z$

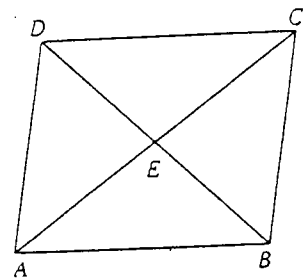


- (A) I only  
 (B) I and II only  
 (C) I and III only  
 (D) II and III only  
 (E) III only

19. If it is given that an altitude of a triangle bisects its base, which of the following can be proved?

- (A) the triangle is isosceles  
 (B) the triangle is equilateral  
 (C) the triangle is a right triangle  
 (D) the triangle has an obtuse angle  
 (E) the triangle has three acute angles

20. In the figure,  $AB \parallel CD$ ,  $AD \parallel BC$ , and  $DC \neq BC$ . Which of the following is NOT a correct conclusion?



- (A)  $\triangle ADC \cong \triangle CBA$   
 (B)  $\angle DCA$  has the same measure as  $\angle BCA$   
 (C)  $\angle ADC$  has the same measure as  $\angle CBA$   
 (D)  $AD = BC$   
 (E)  $DE = BE$

21. The statement  $(p \vee q) \rightarrow p$  is false if

- (A)  $p$  is true and  $q$  is true
- (B)  $p$  is true and  $q$  is false
- (C)  $p$  is false and  $q$  is true
- (D)  $p$  is false and  $q$  is false
- (E) the statement is a tautology

22. Which of the following is equivalent to the statement "Having equal radii is necessary for two circles to have equal areas"?

- I. Having equal areas is sufficient for two circles to have equal radii.
  - II. Two circles have equal areas only if they have equal radii.
  - III. Having equal radii implies that two circles have equal areas:
- (A) only I
  - (B) only III
  - (C) only I and II
  - (D) only II and III
  - (E) I, II, and III

23. Given the statement "If  $x = 2$ , then  $x^2 = 4$ ." The negation of this statement is

- (A)  $x \neq 2$ , and  $x^2 \neq 4$
- (B)  $x = 2$ , and  $x^2 \neq 4$
- (C)  $x \neq 2$  or  $x^2 = 4$
- (D)  $x \neq 2$ , and  $x^2 = 4$
- (E)  $x \neq 2$  or  $x^2 \neq 4$

24. Given these statements:

- I. Some numbers are not prime.
- II. No primes are squares.

If *some* means "at least one," it can be concluded from I and II that

- (A) some numbers are squares
- (B) some squares are not numbers
- (C) some numbers are not squares
- (D) no numbers are squares
- (E) none of the above is a conclusion of I and II

25. In a particular town the following facts are true:

- I. Some smarties do not smoke cigars.
- II. All men smoke cigars.

A necessary conclusion is

- (A) some smarties are men
- (B) some smarties are not men
- (C) no smartie is a man
- (D) some men are not smarties
- (E) no man is a smartie

26. Given the statement "The student will not pass the course only if he does not come to class," which one of the following can be concluded?

- (A) If the student does not pass the course, then he probably missed too many classes.
- (B) If a student comes to class, then he may pass course.
- (C) If a student comes to class, then he will pass the course.
- (D) If a student passes the course, then he came to class.
- (E) If a student does not come to class, then he will not pass the course.

27. The contrapositive of  $p \rightarrow q$  is

- (A)  $p' \rightarrow q$
  - (B)  $p' \rightarrow q'$
  - (C)  $q' \rightarrow p$
  - (D)  $q \rightarrow p'$
  - (E)  $q' \rightarrow p'$
- note  $q'$  means  $\sim q$